Using DOE Strategies in Migrating Existing Designs to New Applications

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Abstract:

Migrating existing designs to new applications can create unexpected risk when their performance sensitivity to system inputs have not been characterized. Typically, designs are assessed solely on margins at a single set point within the design space. Transferring a previously successful design to a new application without having characterized its performance to changes in input variables, however, can result in significant risk exposure and false economies.

This tech brief provides an example of using Design of Experiments (DOEs) strategies in the process of Analysis Leading Design (ALD) to reduce the risk of experiencing the costs of a false economy. An annotated EXCEL spreadsheet is available upon request containing the numerical methods employed to characterize the example design space.

Background:

Figure 1 provides a block diagram of a bracket design which was transferred from a previously successfully application to an engine derivative where the support system, a thin wall case, experienced an HCF failure. To avoid this potential failure mode, the fundamental frequency of the bracket system was required to be above 250 Hz.

The previously successful mount design had been transferred without an understanding of the influence of the hardware's cg location, the mount span, and the case thickness on its fundamental frequency. Due to this lack of insight, significant variations of the mount design had been allowed to populate the series of test engines employed for certification tests. The variation of the fundamental mount frequency, within this population of engines, came to light only after the HCF failure investigation was undertaken. The engine, on which the failure occurred, had a mount height resulting in a roll mode frequency aligned with a high level of vibratory input and which accumulated a significant amount of cycles early in the high imbalance test.

To investigate the failure, a DOE was executed enabling the mount performance to be evaluated throughout the entire existing design space. A simple model provided explanatory power for the observed behavior and the information was employed to drive the redesign effort toward the best solution. The benefit of seeing the entire design space, rather than a single point within it, opened alternative solutions which otherwise would not have been considered. Understanding the entire design space also enabled the team to efficiently determine the root cause of the failure and identify a solution to meet the certification deadline.

Benefits of DOE Strategies:

Quantifying the design space using DOEs enables the performance of a system, at a given design point, to be understood in context. This context provides a means of properly assigning value to changing various design variables to meet requirements. Design decisions, in this process, are based on quantitative rather than qualitative information.

These strategies provide a means for engineers to quantify the influence of design parameters on performance, the proximity of the design set point to a local optimum, the strength of cross-coupling between design variables, and how rapidly performance changes due to input variations. Not only is risk assessed but the potential benefits of making design adjustments are quantified as an existing design is transferred to a similar but not identical application.

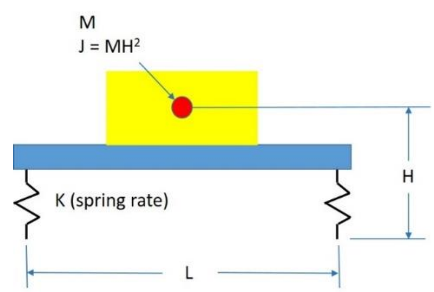

Figure 1

Three primary features governed the fundamental frequency of the failed mount. First was the package cg relative to the case and its resulting inertia. The second was the mount span, and the third was the membrane stiffness of the case.

Had the design space been understood prior to the design being migrated to the derivative engine, not only would the mount itself been incorporated with optimal packaging parameters (L and H) but more importantly the case wall thickness would have been driven towards a more robust design. Once financial resources had been committed to tooling and hardware, the features which could have been used to optimize system performance became instead design constraints. This situation is avoided when ALD is employed.

Three Level Two Factors DOE:

For structural systems, one of the most efficient approaches to understanding the design space of a system is to employ three level two factor DOEs. There are several reasons for this. First, the three levels allow for the curvature of the response surface to be captured and, therefore, an estimate can be made of the proximity of the design to an optimal operating point. Secondly, the potential cross-coupling between the two factors is readily captured, and thirdly these DOEs are extremely efficient to execute and provide a significant amount of information to guide the design.

In the mounting bracket example, however, there are three rather than two factors defining the design space. They are: cg height, bracket span, and case stiffness. This is a common situation where more than two primary factors are required to adequately define the space. Rather than incorporating the third parameter directly into the DOE it can be advantageous to run three separate DOEs holding the third variable constant. This allows the direct influence of the third variable to be quantified and at the same time facilitates understanding the design space associated with the other two variables. The potential downside to this approach is that any cross-coupling between the third factor and the other two design variables cannot be assessed.

There are two common DOE approaches used for three level experiments. The first is the Box-Behnken design and the second is the Central Composite design. Each provides a strategy for fitting second order response surfaces using three-level designs. For this example, the Central Composite design was chosen. This is due to only two primary bracket parameters (L and H) being used in the DOE with the case stiffness being held constant. The Central Composite design provides a full factorial for three levels with two design variables.

Central Composite DOE:

Table 1 provides the definition of the two factor CC DOE. This DOE was executed for minimum, nominal and maximum case membrane thickness.

Table 1 – Central Composite Design

Nominal Case Stiffness		
Run	L	H
	Inches	Inches
1	4.500	4.500
2	4.500	7.500
3	7.500	4.500
4	7.500	7.500
5	3.182	6.000
6	10.607	6.000
7	6.000	3.182
8	6.000	10.607
9	6.000	6.000
10	6.000	6.000
11	6.000	6.000

Response Surface Results:

Since the CC DOE is over determined for a 2nd polynomial, the quality of the fit was checked against a regression coefficient. Equation 1 is the form used to fit the response data and Figure 2 provides the check of the regression analysis.

$$f(L,H) = C_5L^2 + C_4H^2 + C_3L + C_2H + C_1LH + C_0$$

Equation 1

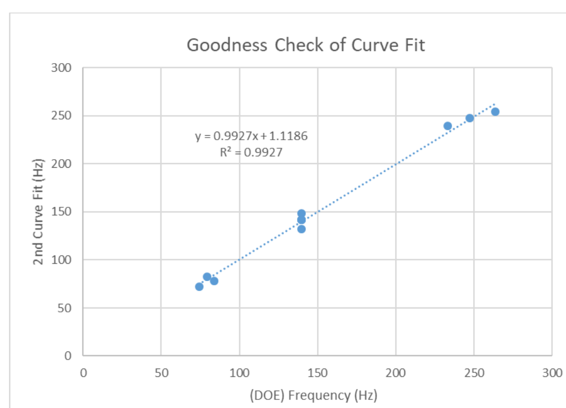

Figure 2

As reflected in the regression coefficient and slope being virtually 1.0, the correlation between the DOE results and the curve fit is extremely good.

Using Design Space Characterizations to Guide Design Decisions:

When using a 2nd degree polynomial to characterize a system, the following parameters can be used to extract performance information regarding the design space:

- *The type of design space surface the polynomial represents*. The surface will be either concave up or down or a saddle. If the surface is concave up or down the stationary point will be a global minimum or maximum design point. If the surface is a saddle it will likely behave as a rising ridge in the actual design space. This is illustrated in the bracket design example.

- *Determine if the location of the stationary point is in the design space.* If the surface is concave up or down, then a potential optimal operating set point exists.

- *The strength of the absolute and relative influence each design parameter has on performance*. This enables the designer to quantify the performance benefits of changing a given design variable.

The response surface characterization is evaluated by the signs of the eigenvalues. If both eigenvalues are positive, then the surface is concave up and the stationary point is a global minimum. If both are negative, then the surface is concave down and the stationary point is a global maximum. If the eigenvalues have different signs, then the surface is a saddle.

In matrix form the response surface can be represented by Equation 2

$$f = C_0 + \boldsymbol{x^T b} + \boldsymbol{x^T B x}$$

Equation 2

Where **B** is:

$$\boldsymbol{B} = \begin{bmatrix} C_5 & \frac{C_1}{2} \\ \frac{C_1}{2} & C_4 \end{bmatrix}$$

The eigenvalues are found by setting the determinant of the matrix to 0.0.

$$\mathbf{0.0} = \begin{vmatrix} C_5 - \lambda & \frac{C_1}{2} \\ \frac{C_1}{2} & C_4 - \lambda \end{vmatrix}$$

For a case spring rate of 20,000 lbs./in, which represents the nominal case thickness, the response surface equation for the bracket mount system is:

$$f(L,H) = -0.22L^2 + 3.63H^2 + 51.62L - 48.36H - 4.14LH + 148.9$$

Equation 3

For this example, the eigenvalues are -1.12 and 4.54 indicating that the global surface is a saddle. Its stationary point is outside the DOE space. Figures 3 and 4 provide plots of the design space.

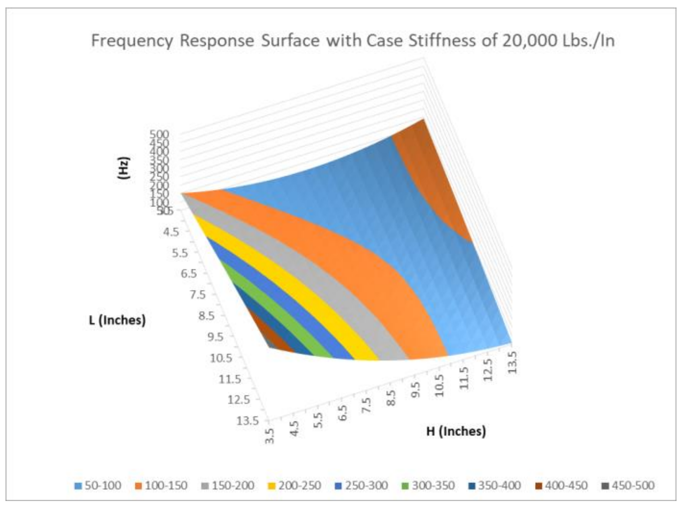

Figure 3

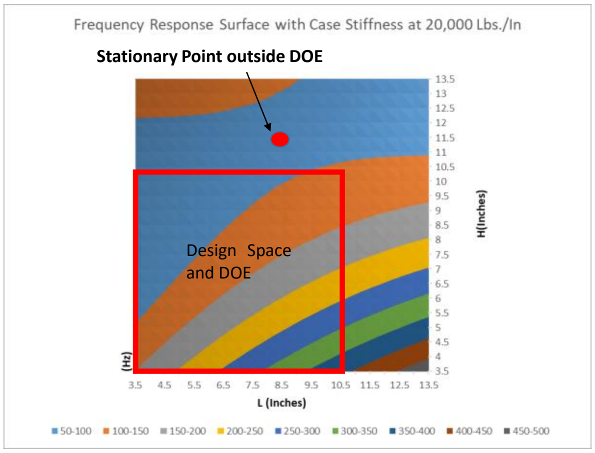

Figure 4

The portion of the surface in the actual design space is a rising ridge. As the cg is lowered and the mount span becomes higher the frequency of the fundamental mode increases. Typically, structural problems tend to be best represented by a rising ridge.

As can readily be seen from the contours bands, the strength of influence from the mount span and the cg height are close in magnitude but opposite in sign. The reason for this is that the fundamental mode is roll participation. The increase in span provides greater leverage against roll motion and the inertia is lowered as the cg is moved closer to the case. Numerically, the influence of the design variables can be quantified by taking the partial derivatives of the response surface with respect to each parameter at the center point of the DOE.

$$\frac{\partial f}{\partial L} = -0.44L - 4.14H + 51.61 = 24.0 \; at \; (6{,}6)$$

Equation 4

$$\frac{\partial f}{\partial H} = 7.27H - 4.14L - 48.36 = -29.6 \; at \; (6{,}6)$$

Equation 5

Solving these two equations simultaneously also provides the location of the stationary point on the surface. Since the frequency response is represented as a saddle, the stationary point is neither a minimum nor maximum. Optimizing the system performance will be found along the boundaries of the design space.

Since the same Central Composite DOE was performed for three case stiffness values, design functions at 250 Hz were created for each case thickness. Figure 5 provides plots of the design parameters producing a 250 Hz fundamental mode frequency for each case thickness.

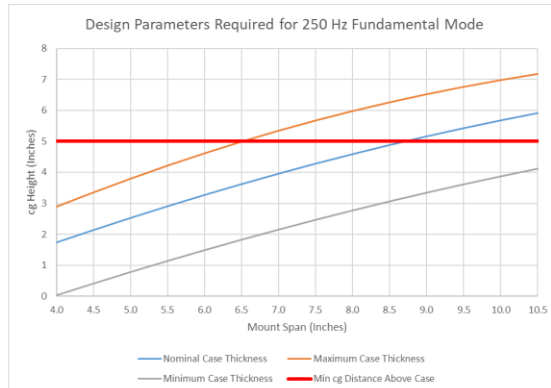

Figure 5

The height of the hardware mounted to the bracket and the potential cording on the circumference of the case required the cg be a minimum of 5 inches above the case. With this constraint, the 250 Hz minimum requirement is not in the design space with a minimum case thickness. The benefit of increasing the case thickness is seen in the reduction in bracket span to meet the performance criterion of 250 Hz. With multiple external packages mounted to the case and the potential weight savings associated with reduced bracket spans, it becomes apparent that attempting to weight optimize the system by only considering the case thickness could be a design fallacy and result in a false economy due to the failure to meet design requirements.

Conclusions:

In the decision-making process, response surface strategies are an efficient means of creating the context required to properly assign value to design parameters used in obtaining a viable solution. Deconstructing design space definitions into series of two parameter design variable surfaces provides an excellent means of numerically and visually extracting information to drive towards successfully migrated designs.